

# Energy dependence of Cronin momentum in saturation model for $p + A$ and $A + A$ collisions

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**Abstract.** The energy dependence of the Cronin momentum for  $p + A$  and  $A + A$  collisions in the saturation model are calculated. This dependence is consistent with simple dimensional considerations and can be used to test the validity of the saturation model. It gives the possibility to distinguish the different variants of the saturation model with precise experimental data and to measure the  $x$  dependence of the saturation momentum.

## 1 Introduction

The Cronin effect [1] (i.e., the observation that the ratio of the yield of particles in  $p + A$  and  $A + A$  collisions to the one in  $p + p$  collisions has a maximum at some intermediate transverse momentum) is one of the remarkable effects in high energy nuclear collisions. Both the saturation model [2] and pQCD describe this behavior well. It is possible that these models can be distinguished by means of some subtle prediction for the Cronin effect. Since there are also many variants of the saturation model we should consider this case first. The first issue here is to choose a parameter for prediction. There are three parameters in the Cronin effect that we can measure: the momentum where the Cronin ratio has a maximum (let us call this the Cronin momentum  $q_C$ ), the value of the maximum of  $R_C$ , and the momentum where the Cronin ratio is unity,  $q_u$ . We know that the value of the Cronin ratio in  $A + A$  collisions has some normalization uncertainty (theoretical and experimental), and therefore the parameters  $R_C$ ,  $q_u$  are not good ones to make predictions. At the same time the Cronin momentum  $q_C$  does not depend on the normalization and therefore is the best candidate for this purpose. The second issue is to choose the appropriate kinematical range. It is well known that the saturation model works well only in the mid-rapidity region and therefore let us consider central rapidity  $p + A$  collisions only. There is only one semihard scale in the saturation model which governs the momentum dependence of the differential cross-section  $\frac{d\sigma_{pA}}{dydq^2}$  (here  $y$  is the rapidity and  $q$  is the transverse momentum of the particles produced). This scale is the saturation momentum  $Q_s$ . Since we have only one semihard scale the Cronin momentum can only depend on this scale. Dimen-

sional considerations give us the only possible choice for the equation which relates  $Q_s$  and  $q_C$ :

$$q_C = \beta Q_s, \quad (1)$$

where  $\beta$  is some dimensionless constant. It is well known that the saturation momentum  $Q_s$  is not a constant. It depends on the Bjorken variable  $x$  which in this process is defined by the simple formula

$$x = \frac{q}{\sqrt{s}}. \quad (2)$$

Since  $q_C$  is the only known momentum therefore, instead of (1) we will have

$$q_C = \beta Q_s \left( \frac{\beta_1 q_C}{\sqrt{s}} \right), \quad (3)$$

where  $\beta_1$  is another dimensionless constant.

$Q_s(x)$  can easily be defined. Using the geometric scaling effect for small  $x$  we have

$$Q_s^2(x) = A^{1/3} Q_{s0}^2 \left( \frac{x_0}{x} \right)^\lambda, \quad (4)$$

where  $\lambda = 0.3$  is the geometric scaling constant and  $Q_{s0}, x_0$  are some parameters whose exact values can be defined using the fact that  $Q_s$  is equal to 1–2 GeV in a reaction with an Au nucleus at  $\sqrt{s} = 200$  GeV. Therefore (3) can be solved easily. It results in the following expression for  $q_C$ :

$$q_C = q_C^0 A^{\frac{1}{3(2+\lambda)}} \sqrt{s}^{\frac{\lambda}{2+\lambda}}. \quad (5)$$

If we take logarithms of both parts we have

$$\ln(q_C) = a + b \ln(\sqrt{s}), \quad (6)$$

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where the parameters  $a$  and  $b$  are defined by

$$a = \frac{1}{3(2+\lambda)} \ln(A) + \ln(q_C^0), \quad (7)$$

$$b = \frac{\lambda}{2+\lambda} = 0.1304.$$

Equation (7) for the parameters  $a$  and  $b$  is based on the fact that  $Q_s$  is the only momentum scale in the process. However, a soft scale  $\Lambda_{\text{QCD}}$  exists and it is obvious that its existence will change (6). Therefore, we should calculate the Cronin momentum numerically for different energies and check the validity of (6).

## 2 Cronin ratio in the saturation model

Let us consider the Cronin ratio for  $p+A$  collisions (in this article we make all calculations only for the Au nucleus)

$$R_{pA} = \frac{\frac{d\sigma^{pA}}{dyd^2q}}{A \frac{d\sigma^{pp}}{dyd^2q}}. \quad (8)$$

As we stated before we consider the central rapidity region only (i.e.  $y = 0$ ). In the saturation model the gluon production cross-section can be expressed as

$$\frac{d\sigma^{pA}}{d^2q dy} = \frac{2\alpha_s}{C_F} \frac{1}{q^2} \int d^2k \phi_p(x_1, q^2) \phi_A(x_2, (q-k)^2), \quad (9)$$

where  $\phi_{A,p}$  is the unintegrated gluon distribution of the nucleus and proton, and  $x_1, x_2$  are defined by

$$x_1 = \frac{q}{\sqrt{s}} e^{-y}, \quad x_2 = \frac{q}{\sqrt{s}} e^y. \quad (10)$$

In leading logarithmic order we can rewrite (9) in the following form [4]:

$$\frac{d\sigma^{pA}}{d^2q dy} = \frac{2\alpha_s}{C_F} \frac{1}{q^2} \times (x_1 G_p(x_1, q^2) \phi_A(x_2, q^2) + x_2 G_A(x_2, q^2) \phi_p(x_1, q^2)), \quad (11)$$

where  $xG(x, q^2)$  is the gluon distribution function which is related to  $\phi(x, k^2)$  by the following formula:

$$xG(x, q^2) = \int_A^q \phi(x, k^2) dk^2. \quad (12)$$

The same formula can be applied to  $p+p$  collisions,

$$\frac{d\sigma^{pp}}{d^2q dy} = \frac{2\alpha_s}{C_F} \frac{1}{q^2} \times (x_1 G_p(x_1, q^2) \phi_p(x_2, q^2) + x_2 G_p(x_2, q^2) \phi_p(x_1, q^2)). \quad (13)$$

Let us suppose that in the unintegrated gluon distribution function of the proton  $\phi_p(x, q^2)$  does not depend on  $x$  considered the kinematical region and that

$$\phi_p(x, q^2) = \frac{\alpha_s C_F}{\pi} \frac{1}{q^2}. \quad (14)$$

Then the following relation can be written for the Cronin ratio:

$$R_{pA} = \frac{1}{A} \left( \frac{\phi_A(x_2, q^2)}{\phi_p(x_2, q^2)} + \frac{G_A(x_2, q^2)}{G_p(x_2, q^2)} \right), \quad (15)$$

or, since we suppose we have the central rapidity region,

$$R_{pA} = \frac{1}{A} \left( \frac{\phi_A(x, q^2)}{\phi_p(x, q^2)} + \frac{G_A(x, q^2)}{G_p(x, q^2)} \right), \quad (16)$$

where  $x = \frac{q}{\sqrt{s}}$ .

All we need now is the expression for the unintegrated gluon distribution function. Let us consider three models for the gluon distribution function: the Kharzeev–Levin–Nardi model proposed in [3], the ‘‘dipole’’ model and McLerran–Venugopalan model proposed in [5, 6]. It should be mentioned that the ‘‘dipole’’ model is the only one for which a theoretical proof [11] for (9) exists. Nevertheless we make calculations for all of them.

## 3 Kharzeev–Levin–Nardi model

Let us consider the simplest form of this model. The unintegrated gluon distribution function  $\phi(x, q^2)$  can be written as

$$\phi_A(x, q^2) = \phi_A^0, \quad q < Q_s(x), \quad (17)$$

$$\phi_A(x, q^2) = \phi_A^0 \frac{Q_s^2(x)}{q^2}, \quad q > Q_s(x),$$

where  $\phi_A^0$  is a normalization factor.

Therefore we will have for the gluon distribution function  $G(x, q^2)$  the following expression:

$$xG_A(x, q^2) = \phi_A^0 (q^2 - \Lambda_{\text{QCD}}^2) \quad (18)$$

for  $q < Q_s(x)$  and

$$xG_A(x, q^2) = Q_s^2(x) \phi_A^0 \left( \ln \left( \frac{q^2}{Q_s^2(x)} \right) + 1 - \frac{\Lambda_{\text{QCD}}^2}{Q_s^2(x)} \right) \quad (19)$$

for  $q > Q_s(x)$ .

Then we will have the following expression for the Cronin ratio:

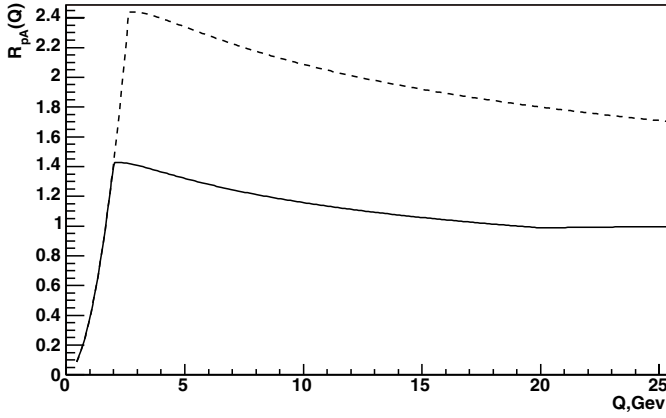
$$R_{pA} = \frac{\phi_A^0 \pi}{A \alpha_s C_F} \left( q^2 + \frac{q^2 - \Lambda_{\text{QCD}}^2}{\ln \left( \frac{q^2}{\Lambda_{\text{QCD}}^2} \right)} \right) \quad (20)$$

for  $q < Q_s(x)$  and

$$R_{pA} = \frac{\phi_A^0 \pi Q_s^2(x)}{A \alpha_s C_F} \left( 1 + \frac{\ln \left( \frac{q^2}{Q_s^2(x)} \right) + \left( 1 - \frac{\Lambda_{\text{QCD}}^2}{Q_s^2(x)} \right)}{\ln \left( \frac{q^2}{\Lambda_{\text{QCD}}^2} \right)} \right) \quad (21)$$

for  $q > Q_s(x)$ .

If we look at (20) and (21) we see that the Cronin ratio  $R_{pA}$  here is a non-decreasing function of the momentum



**Fig. 1.** Cronin ratio for Kharzeev–Levin–Nardi gluon distribution function for  $p + A$  collisions  $\sqrt{s} = 200$  GeV (solid curve) and  $\sqrt{s} = 1700$  GeV (dashed curve)

$q$ , and therefore it is not clear if there is any Cronin like behavior in this model. However, it should be mentioned that  $x$  depends on  $q$  by the relation  $x = \frac{q}{\sqrt{s}}$ , and, since we have (4),  $R_{pA}$  has a maximum at some momentum  $q_C$  (Fig. 1) which value is *approximately* defined by the following equation:

$$q_C = Q_s \left( \frac{q_C}{\sqrt{s}} \right) \tag{22}$$

and is modified slightly by the logarithmic terms in (21). Nevertheless, we have a  $q_C$  energy dependence similar to (6) with slope  $b = 0.1042$ .

### 4 “Dipole” model

In the “dipole” model we can relate the unintegrated gluon distribution function to the gluon dipole cross-section. This was done in [9,11] and the expression for the unintegrated gluon distribution function can be written as (let us suppose that the nucleus is cylindrical)

$$\phi_A(x, q^2) = \frac{4S_A C_F}{\alpha_s (2\pi)^3} \int d^2r e^{-iqr} \nabla_r^2 N_G(r, x) \tag{23}$$

or

$$\phi_A(x, q^2) = \frac{4S_A C_F}{\alpha_s (2\pi)^2} \int dr J_0(qr) r \nabla_r^2 N_G(r, x), \tag{24}$$

where  $J_0(x)$  is Bessel function.

We will have the following relation for the gluon distribution function  $G(x, q^2)$ :

$$xG_A(x, q^2) = \frac{8S_A C_F}{\alpha_s (2\pi)^2} \int dr k J_1(kr) \Big|_{k=A}^{k=q} \nabla_r^2 N_G(r, x). \tag{25}$$

It is well known that the Balitsky–Kovchegov [8] equation defines the  $x$ -behavior of the dipole scattering cross-section  $N_G(r, x)$ . Since it is not solved analytically for now we will

not use it here. It was shown in [10] that using  $N_G(r, x)$  from the Balitsky–Kovchegov equation lowers  $R_{pA}$  when  $\sqrt{s}$  increases. Therefore, it is interesting to check the validity of (6) in this case. These calculations can be found elsewhere [12] and we choose a simpler way here.

Let us define ad hoc that

$$N_G(r, x) = 1 - e^{-r^2 Q_s(x)^2 \ln(1/rA)/4}, \tag{26}$$

i.e. we put all  $x$  dependence in the  $Q_s(x)$  definition. It is obvious that we cannot use (26) directly since  $N_G(r, x)$  does not have a very good behavior for large  $r$  (i.e. if  $r \rightarrow \infty$  then  $N_G(r, x)$  becomes negative instead of unity). Therefore, we should regularize (26) somehow. Let us regularize  $N_G(r, x)$  by the following prescription:

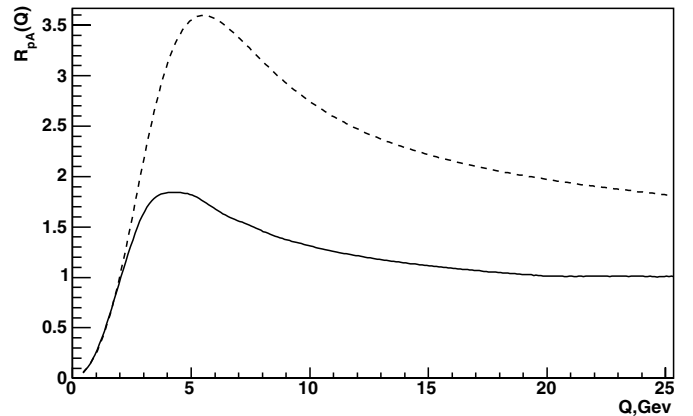
$$N_G(r, x) = 1 - e^{r^2 Q_s(x)^2 (\ln(rA) - \sqrt{(\ln(rA))^2 + \epsilon^2}) + \ln(r_0 A)) / 8}, \tag{27}$$

and set  $r_0 = \frac{1}{\sqrt{\epsilon A}}$  and  $\epsilon < 1$ . (If  $\epsilon$  is less than unity then the result does not depend on its exact value.) It should be noted that the final result does not depend on the regularization scheme and we could regularize  $N_G(r, x)$  with a simpler prescription:

$$\begin{aligned} N_G(r, x) &= 1 - e^{-r^2 Q_s(x)^2 \ln(1/rA)/4}, & r < r_0, \\ N_G(r, x) &= 1 - e^{-r^2 Q_s(x)^2 \ln(1/r_0 A)/4}, & r > r_0, \end{aligned} \tag{28}$$

but this regularization is inconvenient since  $\nabla_r^2 N_G(r, x)$  is singular, and we should also regularize  $\nabla_r^2 N_G(r, x)$  itself.

Using the regularized  $N_G(r, x)$  and (24) and (25) the Cronin ratio can be calculated numerically. The momentum dependence of the Cronin ratio (16) is presented in Fig. 2. The position of the maximum (i.e. the Cronin momentum  $q_C$ ) can easily be calculated numerically for different energies using this dependence. The result is presented in Fig. 4. The slope calculated by the fitting procedure is equal to  $b = 0.1120$ . It should be mentioned that the lines have a different position, but they have almost the same slope, *approximately* equal to the one calculated in (7).



**Fig. 2.** Cronin ratio for the “dipole” gluon distribution function for  $p + A$  collisions for  $\sqrt{s} = 200$  GeV (solid curve) and  $\sqrt{s} = 1700$  GeV (dashed curve)

## 5 McLerran–Venugopalan model

In the McLerran–Venugopalan model the expression for the unintegrated gluon distribution function was found in [5,6], and it can be written as

$$\begin{aligned} \phi_A(x, q^2) & \quad (29) \\ &= \frac{4C_F}{\alpha_s(2\pi)^3} \int d^2b d^2r \frac{e^{-iqr}}{r^2} \left(1 - e^{-r^2 Q_s^2 \ln(\frac{1}{r\Lambda})/4}\right). \end{aligned}$$

If we will consider a cylindrical nucleus we have

$$\phi_A(x, q^2) = \frac{4S_A C_F}{\alpha_s(2\pi)^3} \int d^2r \frac{e^{-iqr}}{r^2} \left(1 - e^{-r^2 Q_s^2 \ln(\frac{1}{r\Lambda})/4}\right), \quad (30)$$

or

$$\phi_A(x, q^2) = \frac{4S_A C_F}{\alpha_s(2\pi)^2} \int dr \frac{J_0(qr)}{r} \left(1 - e^{-r^2 Q_s^2 \ln(\frac{1}{r\Lambda})/4}\right). \quad (31)$$

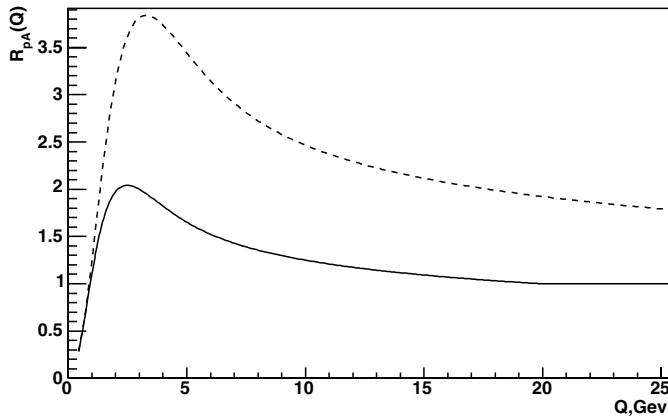
However, it is better to use the expression proposed in [7] which relates the unintegrated gluon distribution function in the McLerran–Venugopalan model and the forward scattering amplitude  $N_G(r, x)$  of a gluon dipole on a nucleus. According to [7] we will have for the gluon distribution function

$$\phi_A(x, q^2) = \frac{4S_A C_F}{\alpha_s(2\pi)^2} \int dr J_0(qr) \frac{1}{r} N_G(r, x). \quad (32)$$

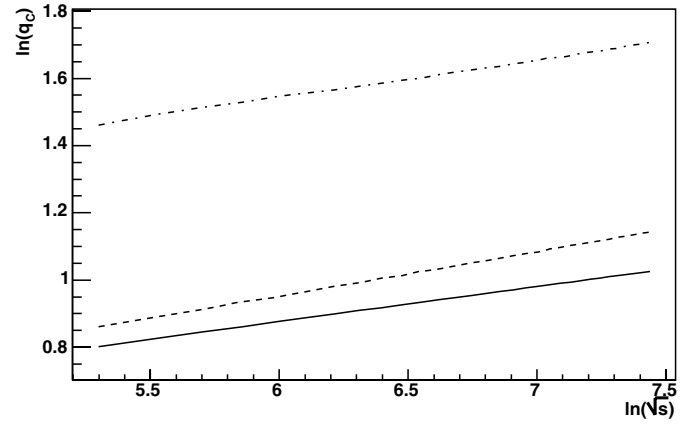
A similar expression can be written for the gluon distribution function  $G(x, q^2)$ :

$$xG_A(x, q^2) = \frac{8S_A C_F}{\alpha_s(2\pi)^3} \int dr \frac{k J_1(kr)|_{k=q}}{r^2} N_G(r, x). \quad (33)$$

Like in the previous model the value of the Cronin ratio and the Cronin momentum  $q_C$  can easily be calculated numerically. The result is presented in Figs.3 and 4. Like in the previous models we have a Cronin momentum energy dependence similar to (6) with slope  $b = 0.1323$ .



**Fig. 3.** Cronin ratio for the McLerran–Venugopalan gluon distribution function for  $p + A$  collisions for  $\sqrt{s} = 200$  GeV (solid curve) and  $\sqrt{s} = 1700$  GeV (dashed curve)



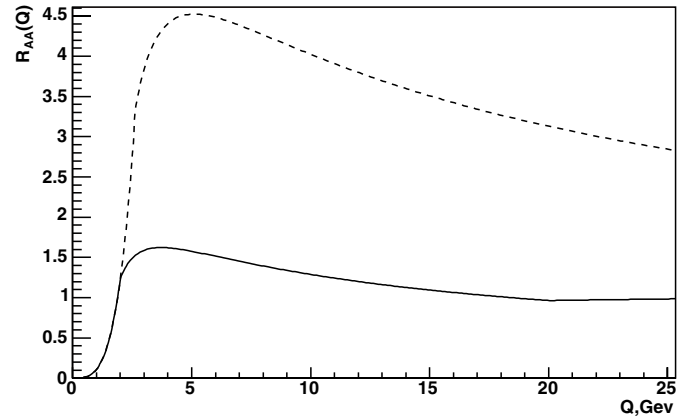
**Fig. 4.** Dependence of  $\ln(q_C)$  (for  $R_{pA}$ ) on  $\ln(\sqrt{s})$  for different models: Kharzeev–Levin–Nardi (solid curve), McLerran–Venugopalan (dashed curve), ‘dipole’ (dot-dashed curve)

## 6 $A + A$ collisions

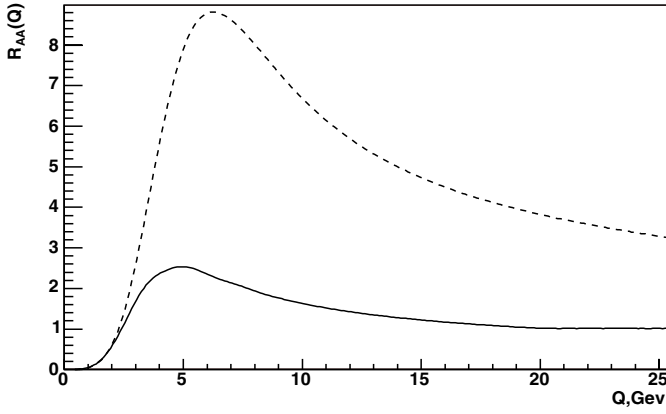
Like in  $p + A$  collision in  $A + A$  collisions (we will consider only the central rapidity region) there is only one semihard scale  $Q_s$ . Therefore, the energy dependence of the Cronin momentum  $q_C$  should be governed by the same relation (3). We can apply all formulas above to this case, since we have the following approximate relation for the Cronin ratio, which is similar to (16):

$$R_{AA} = \frac{G_A(x, p) \phi_A(x, p)}{G_p(x, p) \phi_p(x, p)}. \quad (34)$$

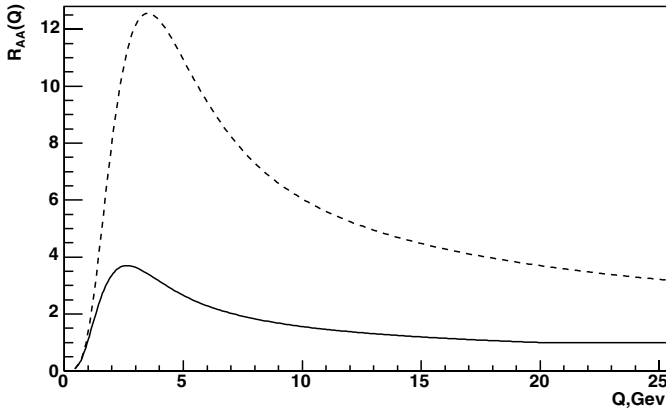
Calculating numerically the  $q$  dependence of the Cronin ratio for the models considered (Figs. 5, 6, and 7) at different energies we have the same linear behavior for  $\ln(q_C)$  as before (Fig. 8) and also have slopes consistent with (7). All data summarized in Table 1 (for the ‘dipole’ model only points with  $\sqrt{s} > 500$  GeV were taken for the slope calculation).



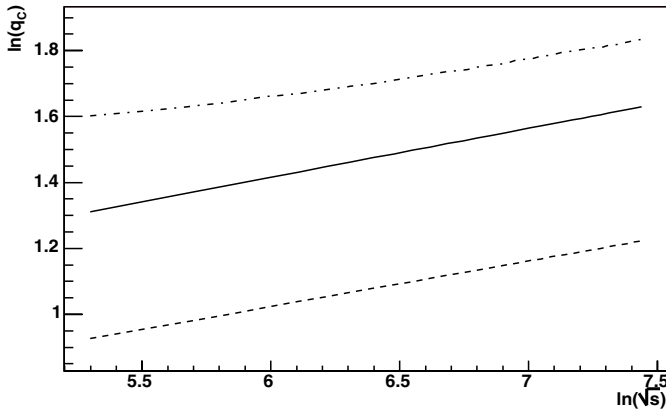
**Fig. 5.** Cronin ratio for the Kharzeev–Levin–Nardi gluon distribution function for  $A + A$  collisions  $\sqrt{s} = 200$  GeV (solid curve) and  $\sqrt{s} = 1700$  GeV (dashed curve)



**Fig. 6.** Cronin ratio for the “dipole” gluon distribution function for  $A + A$  collisions for  $\sqrt{s} = 200$  GeV (solid curve) and  $\sqrt{s} = 1700$  GeV (dashed curve)



**Fig. 7.** Cronin ratio for the McLerran–Venugopalan gluon distribution function for  $A + A$  collisions for  $\sqrt{s} = 200$  GeV (solid curve) and  $\sqrt{s} = 1700$  GeV (dashed curve)



**Fig. 8.** Dependence of  $\ln(q_C)$ (for  $R_{AA}$ ) on  $\ln(\sqrt{s})$  for different models: Kharzeev–Levin–Nardi (solid curve), McLerran–Venugopalan (dashed curve), “dipole” (dot-dashed curve)

**Table 1.** Summary of slopes for different models in  $p + A$  and  $A + A$  collisions

Model	$p + A$	$A + A$
Kharzeev–Levin–Nardi	0.1042	0.1485
McLerran–Venugopalan	0.1323	0.1383
“Dipole”	0.1120	0.1244

## 7 Conclusion

We calculate the energy dependence of the Cronin momentum in several saturation-based models and show that this dependence is consistent with a simple formula based on geometric scaling only. This subtle prediction can be used to test the validity of the saturation model. Since the slope values are slightly different among different variants of the saturation model we have the possibility to distinguish them. This requires more precise measurements of the Cronin effect (at least in the middle momentum region) than those we have for now. These measurements can in turn lead to a precise measurement of the  $x$  dependence of the saturation momentum  $Q_s(x)$ . Since we do not have a precise enough measurement of the Cronin effect for now it is important to consider additional cases (these calculations can be found elsewhere [12]), since differences between the models could become more noticeable:

- (1) It is well known that the dipole scattering cross-section  $N_G(r, x)$  which we set here in simple way can be calculated on a solid theoretical basis using the Balitsky–Kovchegov [8]. It is shown in [10] that using  $N_G(r, x)$  from the Balitsky–Kovchegov equation lowers  $R_{pA}$  when  $\sqrt{s}$  increases so it is important to check the validity of our results in this case.
- (2) In this article we consider the central rapidity region only. This simplification sets a very simple dependence of the Cronin momentum since we have only one semihard scale. In the non-central rapidity region there are at least two semihard scales (in  $A + A$  collisions) and the existence of a second scale can change the result drastically.
- (3) In the case of non-symmetric heavy nucleus collisions (i.e.  $d + Au$ ) we can have the situation when in non-central nucleus collisions the two scales become equal and the Cronin momentum dependence is similar to the  $A + A$  case.
- (4) It is also important to check the validity of our results when we consider different centralities, since when we will put the centrality dependence into account we have a continuous range of scales in the saturation model.

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## References

1. J.W. Cronin, H.J. Frisch, M.J. Shochet, J.P. Boymond, R. Mermod, P.A. Piroué, R.L. Sumner, Phys. Rev. D **11**, 3105 (1975)
2. R. Baier, A. Kovner, U.A. Wiedemann, Phys. Rev. D **68**, 054009 (2003) [hep-ph/0305265]
3. D. Kharzeev, E. Levin, M. Nardi, hep-ph/0212316
4. L.V. Gribov, E.M. Levin, M.G. Ryskin, Phys. Rep. **100**, 1 (1983)
5. Y.V. Kovchegov, Phys. Rev. D **54**, 5463 (1996) [hep-ph/9605446]; Phys. Rev. D **55**, 5445 (1997) [hep-

- ph/9701229]; J. Jalilian-Marian, A. Kovner, L.D. McLerran, H. Weigert, Phys. Rev. D **55**, 5414 (1997) [hep-ph/9606337]; Phys. Rev. D **55**, 5445 (1997) [hep-ph/9701229]; Phys. Rev. D **55**, 5414 (1997) [hep-ph/9606337]
6. Y.V. Kovchegov, A.H. Mueller, Nucl. Phys. B **529**, 451 (1998) [hep-ph/9802440]
  7. D. Kharzeev, Y.V. Kovchegov, K. Tuchin, hep-ph/0307037
  8. I. Balitsky, Nucl. Phys. B **463**, 99 (1996) [hep-ph/9509348]; Y.V. Kovchegov, Phys. Rev. D **60**, 034008 (1999) [hep-ph/9901281]
  9. M. Braun, Eur. Phys. J. C **16**, 337 (2000) [hep-ph/0001268]; Phys. Lett. B **483**, 105 (2000) [hep-ph/0003003]
  10. J.L. Albacete, N. Armesto, A. Kovner, C.A. Salgado, U.A. Wiedemann, hep-ph/0307179
  11. Y.V. Kovchegov, K. Tuchin, Phys. Rev. D **65**, 074026 (2002) [hep-ph/0111362]
  12. V.A. Abramovsky, N.V. Prikhod'ko, in preparation